The Performance-Enhancing Drug Game

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Doping cases hit record

By Tom Weir, USA TODAY

ATHENS — With a record 24 athletes ousted for drug-related violations, perhaps the biggest upset of the Olympics was no U.S. team member flunked a doping test.

Shawn Crawford: winning the gold on 200 meters
Outline

1) A Brief discussion on Doping history and extent

2) The Performance-Enhancing Drug Game

3) Nash Equilibrium Characteristics

4) Relaxing model assumptions

5) Policy implications and conclusions
Doping history

• Old phenomenon: Philostratus and Galerius reports on doping from ancient Olympics in the third century B.C.

• More Recent doping creativity:
  – Oxygen
  – Strychnine
  – Brandy and cocaine – mixed

• Today: Shift from proving existence of unnatural substances towards proving too high levels of natural substances
  – Testosterone
  – Growth hormones
  – EPO
Doping extent

Sports officials: Significant improvement – less doping now than before.

- Atlanta 96: 2 cases (Andrews, 1998)

- Performance decrease (swimming, athletics – typically force events and female events)

- Improved testing, WADA etc.

Still, recall the situation in Athens, 24 caught, and a lot either caught before the games, or simply not daring to enter
Doping extent - alternative angle

Chicago doctor, Bob Goldman asked 198 US athletes the following questions: (Andrews 1998)

1) Given the choice of taking a drug with certain effect (certain win) and no probability of being caught, what would you do? (99% YES)

2) If the same situation could be repeated over 5 consecutive years, but with a certain death caused by side effects after 5 years, would you still take the drug? (more than 50% YES)
Some relevant previous research:


A simple Game model – assumptions (1)

- 2 athletes (of equal strength) compete against each other in some sports event.

- The athletes’ possible strategic choices are to use dope or not. Hence, we assume only one available drug.

- The drug is assumed effective, that is if one agent takes the drug and the other does not, the "drug-taker" wins the competition with certainty. The drug is also assumed to have equal effect on both athletes. That is, if both athletes takes the drug, they are again equal in strength.

- Both agents must decide (simultaneously) before the competition on whether to take the drug or not, and this decision is made only once ("one-shot").
A simple Game model – assumptions (2)

- The agents pay-offs are defined according to three interesting outcomes for each agent:
  - $W$: Agent $i$ wins the competition
  - $L$: Agent $i$ looses the competition
  - $E$: Agent $i$ is exposed as a drug abuser

To simplify calculations, without loss of generality*, we define the following utilities for each of the above outcomes:

$$u() = \begin{cases} 
  u_i(W) = a, & a > 0 \\
  u_i(L) = 0 \\
  u_i(E) = -c, & c > 0 
\end{cases} \quad (1)$$

*Note that the above definition implicitly makes the assumption that both agents have a symmetrical utility structure. That is, the value of winning, loosing or being exposed is the same for both agents.
A simple Game model – assumptions (3)

- The probability of being exposed as a drug abuser, \( r \), is assumed to be a "nature call" in this game, that is both players know it and can not in any way influence it, neither the value nor the actual test which takes place after the competition. The probability of being exposed as a drug abuser if drugs are not used, is assumed zero. (Unlike real doping tests, we hence assume "perfectness".)

- Furthermore, we assume – for simplistic reasons* – that the pay-off received by any agent, is kept even if this agent is caught in doping.

- Finally, both agents know all there is to know (every assumption defined above – complete information).

*Even though the latter events in Salt Lake City may prove this to be a fairly realistic assumption.
A Two-Player Simultaneous Game

\begin{align*}
\text{Expected utility (D,D) case:} \\
\frac{1}{2} a + 0 - r \cdot c = \frac{1}{2} a - rc \\
\text{Expected utility (D,ND) case: (AGENT 1)} \\
1 \cdot a + 0 \cdot 0 - r \cdot c = a - rc
\end{align*}
Nash Equilibria (1)

Crucial assumption: Sign of $\frac{1}{2}a - rc$. Reasonably to assume that $\frac{1}{2}a - rc > 0$, no doping would take place if not. Then,

$$\frac{1}{2}a - rc > 0 \quad (4)$$

$$\frac{1}{2}a - rc + \frac{1}{2}a > \frac{1}{2}a \Rightarrow a - rc > \frac{1}{2}a \quad (5)$$
Nash equilibria (2)
Some simple conclusions:

- Everybody use drugs – (D,D) is a unique Nash equilibrium

- Under reasonable assumptions of $r, c > 0$, $\frac{1}{2}a - rc < \frac{1}{2}a$, or the Nash equilibrium (D,D) is of "Prisoner's Dilemma" type. Hence, regulation or anti-doping work is necessary.

- A necessary condition for Efficient anti-doping work is: $\frac{1}{2}a - rc < 0$ or $r > \frac{a}{2c}$. That is, $r$ must be increased sufficiently, unless, a pareto worsening is the effect. (regulators use more money on anti-doping work, same number of "dopers")

- Anti-doping work should be strongly differentiated between sports activities. If $a_{soccer} \gg a_{curling}$ and $c_{soccer} \approx c_{curling}$ then, $r_{soccer} \gg r_{curling}$. 
Relaxing equal strength assumption

Assume now: AGENT1 is better than AGENT2 with common knowledge probability $p$.

\[
\begin{array}{c|cc}
\text{AGENT 1} & \text{D} & \text{ND} \\
\hline
\text{D} & (1-p)a-re & 0 \\
\text{ND} & pa-re & a-re \\
\end{array}
\]

In the (D,D) and (ND,ND) cases, AGENT1 beats AGENT2 with probability $p$ – the drug has still equal effect.

In the (ND,D) and (D,ND) cases, we still assume (to make things simple) that the drug-taker wins with certainty. That is, the drug is ”magical”.
Nash equilibria: unequal strength (1)

AGENT1 better than AGENT2 ⇒ \( p > \frac{1}{2} \).
Consequently: if \( \frac{1}{2}a - rc > 0 \) then,

\[
pa - rc > 0
\]  \( (6) \)

Adding \( (1 - p)a \) to each side of (6) yields:

\[
a - rc > (1 - p)a
\]  \( (7) \)

or

\[
\begin{array}{c|c}
\hline
\text{AGENT 1} & \text{AGENT 2} \\
\hline
D & \text{ND} \\
\hline
pa - rc & 0 \\
(1-p)a - rc & a - rc \\
a - rc & (1-p)a \\
0 & pa \\
\hline
\end{array}
\]
Nash equilibria: unequal strength (2)

The rest of the Best Reply functions are determined by the sign of the expression:

\[(1 - p)a - rc\]  \hspace{1cm} (8)

If \((1 - p)a - rc > 0\) it’s back to the initial case. However, if \((1 - p)a - rc < 0\), No Nash equilibrium in pure strategies exist, hence, a unique Nash equilibrium in mixed strategies is the ”Game Theoretic” prediction.

Consequently, regulation within the boundaries of \((1 - p)a - rc < 0 \Rightarrow\) In practical terms: if an athlete believes strongly enough in the magical effectiveness of a drug, doping can not be fought.*

*It is possible to prove (see paper) that if the assumption of a ”magic drug” is relaxed, an unique Nash equilibrium of (ND, ND)-type may at least exist.
Policy implications and conclusions

- Athletes belief in "magical drug" problem in anti-doping work. Hence, increased "uncertainty of outcome" or increased competition is an interesting and not much discussed anti-doping strategy.

- May also serve as an explanation on why certain uncompetitive sports like athletics has more problems with doping than f.i. soccer.

- Common knowledge on actual doping effects may have similar effects. However, incentive problems here.

- Increasing \( c \), the disutility of being caught, obvious!

Could it be that the sports industry has incentives not to fight doping?