A NOTE ON NEWSVENDOR PROBLEM WITH PRICING

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Abstract: The main purpose of the paper is to present a specific case of dynamic pricing for the newsvendor problem. Firstly, the short overview of a newsvendor problem is given together with references to the selected literature and remarks to its applicability. Then, the dynamic pricing principles are discussed together with references to decision dependent randomness case in stochastic programming. The dynamic pricing problem deals with the determination of selling prices over time for a product whose demand is random and whose supply is fixed. We approach this problem by formulating the newsvendor problem, which is introduced as a single period problem in our case. We focus on the specific features of the demand function assuming decision dependent uniform distribution. We assume that its support size linearly decreases with the increase of the price. Under such assumptions, the model has suitable computational features related to the expectation of the objective function. In addition, possibility to increase the profit by the change of the price may appear. The model formulation allows us to use the MAPLE software for symbolic computations and visualization of results. The test results for the selected data set are visualized at the end of the paper.

Keywords: stochastic programming, newsvendor problem, dynamic pricing, MAPLE.

1 Introduction

Newsvendor problem. By literature resources, so called newsvendor (alternatively newsboy) problem has a long and interesting history that was initialized in 1888 by Edgeworth [3], who developed its idea dealing with a bank cash-flow problem. Porteus [22] can be utilized as a useful review for the basic static case and for its generalizations. The newsvendor problem is based on the idea to control the amount of the item, which should be ordered without knowing a future uncertain demand. We further specify it as a problem over a single-period with controlling of a single continuously divisible item. The objective of this stochastic single-period problem is to determine the ordered quantity for a fixed time period that will maximize the expected total profit. Demand is a random variable that is represented by a probability distribution. It is a simple example of a mathematical programming problem, however, it often serves well for introduction of general principles, see, e.g., [23] and [24].

Dynamic pricing. Recently, there is an increasing development of dynamic pricing strategies and their further applications in industry. There are three supporting factors: (1) the increase in availability of demand data, (2) the ease of changing prices due to new technologies, and (3) the availability of models for analyzing demand data and for dynamic pricing, see [4] for details. So, the dynamic pricing problem concerns on the determination of selling prices as a decision variable over time for a product under the demand and supply constraints. In the optimization models this idea leads to the generalization of traditional models when the price is not further considered constant as it becomes a new decision variable. The authors applied this approach to the lot-size problem, see [18], [5], and [6]. Further ideas and applications can be found in [4]. Recently, we have identified the link of dynamic pricing case to so called decision dependent randomness in stochastic programming, see [17] for details and [7] for the original concept discussion.
Newsvendor with pricing. The first mathematical formulation of a newsvendor model with price effects was given by Whitin [29] in 1955. Mills [14] modified the formulation by an explicit specification of the expected demand as a function of the selling price. We interpret the dynamic pricing problem as a price-setting newsvendor problem, which is useful not only because it yields insights into the optimal solution but also because it leads to additional insights into how pricing recourse affects the actions and profits of a price-setting newsvendor. Therefore, the newsvendor problem with pricing is a well known problem, see Monahan [15] and Petruzzi [19] for its various modifications. We present another modification that is suitable for applications in engineering and for computations utilising software for symbolic computations.

Applicability of newsvendor with pricing. The newsvendor problem is one of the fundamental models in stochastic inventory theory, see [22]. We have identified two more original applications in [21]. The first one is related to university budget planning and the next one is used to build a model for the plant capacity design. Both applications are discussed without pricing. Although the considered static newsvendor problem looks quite simple for engineering applications, it can be extended by introduction of further variables and constraints [21]. In addition, pricing approach can tackle cases where the random parameter probability distribution can change with the change of decision variable, see Novotny [17]. Therefore, we may conclude that the ideas of the paper can be applied. e.g., for an optimal choice of the key design parameters, see [9], [25], and [26] or or the production process control as in continuous casting, see [13], [27], and [28]. In such cases, there are a few key design or control variables that can be often treated similarly as in the newsvendor problem, and may also influence the probability distribution change. Thus, the model discussed in the paper can lead to further modifications that might be useful for modern engineering applications. However, the objective function unimodality appearing in the paper cannot be guaranteed in general, and hence, the heuristics will be needed, see [16], [10], [11], and [12] for various possibilities.

2 Newsvendor problem

Newsvendor model. In this section, we discuss modeling details of the problem of controlling the inventory of a single item with stochastic demand over a single period. This problem is known as the newsvendor problem faced by a newsvendor trying to decide how many newspapers to stock on a newsstand before observing the demand. Our version of the problem is formulated without consideration of salvage cost, and it means that faced by a newsvendor trying to decide how many newspapers to stock on a newsstand before observing the demand. Our version of the problem is formulated without consideration of salvage cost, and it means that therefore the amount of unsold item is thrown away. Therefore, the objective function of the underlying stochastic program is

\[
E \{ f(x, \xi) \} = \int_{-\infty}^{\infty} f(x, t) dF(t) = \int_{\xi \leq t} (p - c)x dF(t) + \int_{\xi > t} (p - c)x dF(t) = (p - c)x - p \int_{\xi \geq t} (x - t) dF(t).
\]

We further assume the bounded support of \( \xi \) i.e. \( P(\xi \in [a, b]) = 1 \), and so, we get

\[
\max_x \{ p \min(x, \xi) - cx \mid x \geq 0 \}.
\]
\[ E_\xi f(x, \xi) = \begin{cases} (p - c)x, & x < a, \\ (p - c)x - p \int_a^x (x - t) dF(t), & x \in [a, b], \\ -cx + pE_\xi, & x > b, \end{cases} \]

where \((p - c)x - p \int_{x > a} (x - t) dF(t) = (p - c)x - px + pE_\xi\), and so, \(E_\xi f(x, \xi) = (p - c)x - pE_\xi (x - \xi)_+\), where \((x - \xi)_+ = \max\{x - \xi, 0\}\).

**Uniform demand.** For the uniform probability distribution \(\xi \sim U(a, b)\) and \(x \in [a, b]\):

\[ E_\xi f(x, \xi) = (p - c)x - p \int_a^x \frac{1}{b - a} dt = (p - c)x - \frac{p(x - a)^2}{2(b - a)}. \]

Then max\{\(E_\xi f(x, \xi)\)\(|x \geq 0\}\} = max\{\(p - c)x - pE_\xi (x - \xi)_+\)\(|x \geq 0\}\} = max\{\(p - c)x - \frac{p(x - a)^2}{2(b - a)}\)\(|x \geq 0\}\}. Therefore \(x_{max} = a + \frac{(b - a)(p - c)}{p}\).

**Example.** Let us show results for the aforementioned model and particular data. The following values for parameters of the model are taken into account: \(p = 15\), \(c = 10\), \(a = 30\) and \(b = 50\). Then the expected objective function is piece-wise defined:

\[ E_\xi f(x, \xi) = \begin{cases} f_1(x) = (p - c)x, & x < a, \\ f_2(x) = (p - c)x - \frac{p(x - a)^2}{2(b - a)}, & x \in [a, b], \\ f_3(x) = -cx + \frac{a+b}{2}, & x > b. \end{cases} \]

Then \(x_{max} = \frac{110}{3}\) and \(E_\xi f(x_{max}, \xi) = 166.5\). The objective function is shown on Figure 1.

![Figure 1: The objective function of the example.](image)

### 3 Newsvendor problem with pricing

**Main idea.** The dynamic pricing problem concerns on the determination of selling prices over time for a product whose demand is random and whose supply is fixed. We approach the problem within the following modeling framework: A producer (retailer) has a single opportunity to establish a capacity (or inventory) level prior to the start of each period in a selling season. At the beginning of each period, a price is also chosen and announced. Then, the demand is realized, the period ends, and the decision situation can be repeated.
Decision dependent randomness. Therefore, since now we have to focus on the fact that \( \xi \) depends on \( p \), so we denote it as \( \xi(p) \). As the decision \( p \) may influence the probability distribution of \( \xi \), we write about the decision dependent randomness. Similarly, as in two-stage stochastic programs (see [24]) we search for the separable case. Therefore, we would like to find the description of \( \xi(p) \) by the function separating \( p \) and a new random variable \( \eta \) that does not depend on \( p \) (i.e. \( \xi = g(p, \eta) \) where \( g : R \times R \rightarrow R \) for a set of real numbers \( R \) and random variable \( \eta \)). Such models are solved in Petruzzi [19] and Monahan [15]. Mostly, the random influence \( \eta \) is introduced in a separable additive way. Another already studied possibility is a multiplicative case discussed by Karlin in [8]. We introduce the combined case where the demand-price dependence is linear, however, coefficients are uniformly distributed. So, we further assume that the random demand \( \xi \) is a linear function of price \( p \xi(p) = \alpha p + \beta \) where \( \alpha \) and \( \beta \) are uniformly distributed parameters, and so \( \xi(p) \). The uniform distribution is suitable for cases where bounds of uncertainty are known, otherwise there is a lack of information about uncertainty. We may think that this linear dependency does not approximate some real situations very well. Some authors use a hyperbolic dependency, see Monahan [15] that can be piece-wise approximated. So, thinking about Taylor expansion features, we assume that linear approximation can be acceptable.

Newsvendor model with pricing. The generalized model is:

\[
\max_{x,p} \{ p \min(x, \xi(p)) - cx \ | \ x \geq 0, \ \frac{\beta}{\alpha} \geq p \geq 0, p \in [p_0, p_1] \}
\]

where the decision variables are \( x \) ordered amount and \( p \) selling price per-unit (\( p > c \) is satisfied implicitly). The random variable is again \( \xi \) single-period random demand and newly \( \alpha \), \( \beta \) are random parameters in the linear demand function (\( \alpha > 0, \beta < 0 \)). The constant parameter is \( c \) buying cost per-unit, \( p_0 \) and \( p_1 \) are real bounds for the choice of the price \( p \) and \( p_0 < p_1 \). Let us further consider the continuous uniform probability distribution as above. Then the expected profit can be formally written as follows if we assume that \( t(p) \) and \( dF(t(p)) \) denote parametric dependence of integral on \( p \)

\[
E_{\xi(p)} f(x, p, \xi(p)) = \int_{-\infty}^{\infty} f(x, p, t(p))dF(t(p)) = \\
\int_{x \geq t(p)} (p(t(p) - cx))dF(t(p)) + \int_{x < t(p)} (px - cx)dF(t(p)) = (p - c)x - p \int_{x \geq t(p)} (x - t(p))dF(t(p)).
\]

We know that \( \xi(p) \in [a(p), b(p)] \) where \( a(p) = \alpha_1 p + \beta_1 \) and \( b(p) = \alpha_2 p + \beta_2 \) where \( \alpha_1, \beta_1, \alpha_2, \beta_2 \in R \), and so, we may rewrite the model as follows

\[
E_{\xi(p)} f(x, p, \xi(p)) = \begin{cases} 
(p - c)x, & x < a(p), \\
(p - c)x - \frac{\alpha_1}{\beta_1} \int_{a(p)}^{b(p)}(x - t(p))dF(t(p)), & x \in [a(p), b(p)], \\
-cx + pE\xi(p), & x > b(p),
\end{cases}
\]

where \( (p - c)x - p \int_{x \geq t(p)} (x - t(p))dF(t(p)) = (p - c)x - px + pE\xi(p) \) and so \( E_{\xi(p)} f(x, p, \xi(p)) = (p - c)x - pE\xi(p)(x - t(p))_+ \), where \( (x - t(p))_+ = \max\{x - t(p), 0\} \).

Decision dependent uniform distribution. For \( \alpha \sim U(\alpha_1, \alpha_2) \) and \( \beta \sim U(\beta_1, \beta_2) \) and \( \xi(p) \sim U(a(p), b(p)) \):

\[
E_{\xi(p)} f(x, p, \xi(p)) = (p - c)x - p \int_{a(p)}^{b(p)}(x - t(p))\frac{1}{b(p) - a(p)}dt(p) = (p - c)x - \frac{p(x - a(p))^2}{2(b(p) - a(p))}. \]

Then the following integral computations can be realized by MAPLE software that aids symbolic computations

\[
\max_{x,p} \{ E_{\xi(p)}[f(x, p, \xi(p))] \ | \ x \geq 0, \ -\frac{\beta}{\alpha} \geq p \geq 0, p \in [p_0, p_1] \} = \\
\max_{x,p} \{ (p - c)x - pE\xi(p)(x - \xi(p))_+ \ | \ x \geq 0, \ -\frac{\beta}{\alpha} \geq p \geq 0, p \in [p_0, p_1] \} = \\
\max_{x,p} \{ (p - c)x - \frac{p(x - a(p))^2}{2(b(p) - a(p))} \ | \ x \geq 0, \ -\frac{\beta}{\alpha} \geq p \geq 0, p \in [p_0, p_1] \}.
\]
Example. Let us take the following values for the model parameters: $p_0 = 10$, $p_1 = 40$, $c = 10$, $a_0 = 45$, $a_1 = 15$, $b_0 = 60$ and $b_1 = 20$ where $a_0 = a(p_0)$, $a_1 = a(p_1)$, $b_0 = b(p_0)$, $b_1 = b(p_1)$ and related $a_1, b_1$ can be derived by common straight line computations. Then, $x_{\text{max}} = 27.29$, $p_{\text{max}} = 32.85$ and $E_{\xi(p)} f(x_{\text{max}}, p_{\text{max}}, \xi(p_{\text{max}})) = 564.8$. Therefore, we can see that by considering the price $p$ as a variable, we achieve the improvement of the profit and this is the illustration of the key idea of introduction of pricing in the model. To illustrate this observation graphically, we further consider the continuous probability distribution analyzed above, $\xi(p) \sim U(a(p), b(p))$, and the aforementioned computations. Then, we obtain piece-wise definition of the objective function:

$$E_{\xi(p)} f(x, p, \xi(p)) = \begin{cases} f_1(x, p) = (p - c)x, & x < a(p), \\ f_2(x, p) = (p - c)\left(x - \frac{p(x - a(p))^2}{2(b(p) - a(p))}\right), & x \in [a(p), b(p)], \\ f_3(x, p) = -cx + \frac{a(p) + b(p)}{2}, & x > b(p). \end{cases}$$

See Figures 2 and 3. There is a set of $[p, x]$ couples describing promising combinations identified by $f_2$ on Figure 2. There is only function $f_2$ on Figure 3 because at this function the objective function maximum is achieved.

**Figure 2:** The domain for the example. **Figure 3:** The objective function for the example.

**Conclusions.** The paper presents a note on the newsvendor problem with pricing. The case of the price dependent random demand is considered. For the uniformly distributed demand, we analyze the case of the support changing linearly. With the use of the software tool for symbolic integration, the explicit mathematical program form is derived and its particular instance is computed and visualized. The applicability of this approach to engineering problems is discussed, and so, it opens possibilities for future research.

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**References**


