An economic model of player trade – a game theoretic approach

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Motivation


“The approach we adopt in this paper is to assume that the firm treats its own contract sales as under its own control, while it assumes that no contract sales take place among other teams.”

Idea: How far can “actual” game theory bring us?
Outline

1) A two-player simultaneous game
2) An arbitrage argument
3) Nash equilibria
4) Proof
5) Conclusions
6) Suggestions for further work
A two-player simultaneous game: Assumptions

i. 2 teams ($T_1, T_2$) engaged in a future match, each team has put up one ("identical") player for sale at a fixed price $c$.

ii. Marginally different playing strengths - $T_1$ slightly "better" than $T_2$ (meaning $[P(\text{Win}), P(\text{Loose}), P(\text{Draw})]=[p+\varepsilon, p-\varepsilon, 1-2p]$ for $T_1$, $\varepsilon > 0$)

iii. Actions for each team, buy the player (B) or not buy (NB)

iv. Buying decisions made simultaneously (or without information on other teams buying decision)

v. Teams maximize Pay off (defined below). A parameter $\alpha$ (assumed small) increases (or decreases) expected point score conditioned on the buying decision. Expected point score calculated by a 3-1-0 award system

vi. A revenue function $R(\text{expected point score})$ is present converting playing strength to cash flow (i.e Pay off = cash flow + $R(\text{expected point score})$

vii. $R''()$ is either $> 0$ or $< 0$, and all information above is known by both teams. (A game of complete information)
A two-player complete information simultaneous game in strategic form

\[(T_1, T_2)-(B, NB): \text{Team 2 sells the player to Team 1. Hence, } T_1 \text{ pays } c \text{ and receives added expected playing strength of } \alpha.\]

**Pay off:**

\[-c + R(3(p+\varepsilon)+1(1-2p)+\alpha) = -c + R(p+3\varepsilon+1+\alpha)\]
An arbitrage argument to establish $c$

- There must be a link between $R()$ and $c$. (i.e. the price should reflect the cash-flow consequence of added playing strength)
- Any team selling a player with the result of a net decrease in Pay-off would be silly
- Assume $\varepsilon=0$ (equal teams) and $T_2$ choose B
  - $T_1$ chooses B $\Rightarrow R(p+1)$
  - $T_1$ chooses NB $\Rightarrow c+R(p+1-\alpha)$ $\Rightarrow R(p+1) = c+R(p+1-\alpha)$
  - Similar arg. for $T_2$ choosing NB $\Rightarrow R(p+1) = -c+R(p+1-\alpha)$

Consequently: 

$$c = \frac{R(p+1+\alpha) - R(p+1-\alpha)}{2}$$
Basic result: Proposition

Given assumptions i) – iv), the two-player simultaneous complete information game in strategic form (previous figure) and the added condition on $c$ (last slide) has two unique Nash equilibria in pure strategies.

- If $R$ is strictly convex, Team 1 buys a player from Team 2.
- If $R$ is strictly concave, Team 2 buys a player from Team 1.
Proof: (Enough to look at $R''() > 0$)

The following inequalities secure a unique pure Nash equilibrium of (B,NB)-type

\[
R(p + 3\varepsilon + 1) > \frac{R(p + 1 + \alpha) - R(p + 1 - \alpha)}{2} + R(p + 3\varepsilon + 1 - \alpha)
\]

\[
R(p + 3\varepsilon + 1) < R(p + 3\varepsilon + 1 + \alpha) - \frac{R(p + 1 + \alpha) - R(p + 1 - \alpha)}{2}
\]

\[
R(p + 3\varepsilon - 1) < R(p - 3\varepsilon + 1 - \alpha) - \frac{R(p + 1 + \alpha) - R(p + 1 - \alpha)}{2}
\]
A little trick to move on....

A linear Taylor expansion: \( R(x \pm \alpha) \approx R(x) \pm \alpha R'(x) \)
and some algebra yields:

\[
0 < \alpha[R'(p + 1 + 3\varepsilon) - R'(p + 1)] \\
0 < \alpha[R'(p + 1 + 3\varepsilon) - R'(p + 1)] \\
0 < \alpha[R'(p + 1) - R'(p + 1 - 3\varepsilon)]
\]

If \( R'' > 0 \), these inequalities are satisfied (QED)
Conclusions: (1)

• The shape of $R$ determines the “trade-direction” completely
• A concave $R$ secures trade by the weaker team (i.e. increased competition)
• Somewhat contradictory to (standard) equilibrium based theory
• However: Note strong assumptions
  • One match (no league)
  • Fairly equal teams ($\varepsilon \approx 0$)
  • Low contribution to team playing strength by trade ($\alpha \ll p+1$)
Conclusions: (2)

• In soccer, single games are important, CL or WC
  – At this level, high quality teams (i.e. $\alpha$ is small)
  – Fairly equal (quality) (i.e. $\varepsilon$ is small)
• Hence, this model (in such situations) predicts an “invisible hand” preserving competition given the “normal” assumption of concave $R$. 
Further work?

- More teams
- More games (league)
- Incomplete information
- Situations other than sports (other labour markets): leisure industry