NUMERICAL COMPUTATION OF MARKOV PERFECT EQUILIBRIA IN A GAME BETWEEN A SPONSOR AND A BUREAU

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One period utility functions

Sponsor: $U(Q,B)=\ln(Q)-\beta_1 B^2$

Bureau: $V(Q,S)=\ln(Q)+\beta_2 \ln(S)$

Sponsor: $U_Q>0, U_B<0$

Bureau: $V_Q>0, V_S>0$
"Production function":

\[ Q = B - S \]
MAJOR REFERENCES


$S_{T-1}(B_{T-2})$ solves

$\underset{S}{\text{Max}} \ V(B_{T-2}, S)$

$B_{T-2}(S_{T-3})$ solves

$\underset{B}{\text{Max}} \ U(B, S_{T-3}) + \delta \cdot U(B, S_{T-1}^*(B))$

$S_{T-3}(B_{T-4})$ solves

$\underset{S}{\text{Max}} \ V(B_{T-4}, S) + \delta \cdot V(B_{T-2}^*(S), S) + \delta^2 \cdot V(B_{T-2}^*(S), S_{T-1}^*(B_{T-2}^*(S)))$

\ldots

\ldots

$S_{T-5}$  $S_{T-3}$  $S_{T-1}$

$B_{T-4}$  $B_{T-2}$
\[
\begin{array}{cccc}
\% & 3.704 & 3.680 & 3.676 & 3.110 & T = 0.90 \\
\% & 3.742 & 3.672 & 3.375 & T = 0.80 \\
\% & 3.834 & 3.654 & 3.452 & T = 0.70 \\
\% & 4.281 & 3.558 & 3.424 & T = 0.50 \\
\% & 4.481 & 3.482 & 3.841 & T = 0.30 \\
\% & 4.513 & 3.437 & 3.817 & T = 0.20 \\
\% & 4.753 & 3.388 & 3.807 & T = 0.10 \\
\% & 4.876 & 3.361 & 3.796 & T = 0.05 \\
\% & 3.333 & 3.333 & - & 3.333 & T = 0.00 \\
\hline
\text{Max. Error} & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[
MPE: q_t = q_{t+1} = q_{t+2} = \frac{3 \cdot T_p}{p} = 0
\]

European Economic Review 31 - 1987
A Theory of Dynamic Oligopoly: III - Enco Maskin & Jean Tirole

TESTING THE NUMERICAL MODEL AGAINST THEORETICAL VALUES FROM:
RESULTS

- The Markov perfect equilibrium solution will generally yield lower output and higher slack than the Nash-Cournot equilibrium solution.

- This yields lower output than what is socially optimal.

- The gap between the Markov perfect equilibrium solution and the Nash-Cournot equilibrium solution, widens as the agents' discount factors increase.

- In the Markov setting, high discount factors (low rates of return) induce the agents to apply aggressive strategies, leading to outcomes with low utility for both.
Figure 5: Budget (B) as a function of the discount factor

\[ \beta_2 = 0.1, \ \beta_1 = 0.2 \]

- Subgame perfect eq.
- Nash/Cournot
- Stackelberg-Sponsor leader
Figure 6: Slack ($S$) as a function of the discount factor

$\beta_2=0.1$, $\beta_1=0.2$

Legend:
- Subgame perfect eq.
- Nash/Cournot
- Stackelberg-Bureau leader
Figure 7:
The Subgame perfect equilibria

Budget (B), as a function of the parameter $\beta_1$ and the discount factor ($\beta_2=0.1$)

- Stackelberg - Bureau leader
- Stackelberg - Sponsor leader
- Nash/Cournot
- Subgame perfect: discount fac. $\in [0.1, 0.3, 0.5, 0.7]$
Figure 10:
The Subgame perfect equilibria
Slack (S), as a function of the parameter $\beta_2$ and the discount factor ($\beta_1=0.1$)
Figure 12: The Subgame perfect equilibria

Q, as a function of the parameter $\beta_2$ and the discount factor ($\beta_1=0.1$)

- Stackelberg - Bureau leader
- Stackelberg - Sponsor leader
- Nash/Cournot
- Subgame perfect: discount fac. $\in [1, 3, 5, 7]$
Figure 13: Budget (B) as a function of both discount factors

\[ \beta_2 = 0.1, \beta_1 = 0.2 \]
Figure 14: Slack (S) as a function of both discount factors.
Figure 15: Production (c) as a function of both discount factors